Bicooperative fuzzy games and solution concepts

Student ID 90192101 Inuiguchi Laboratory NAKAGOSHI Naoto

1. Introduction

When multiple decision makers are regard as one player, it is probable that some of them approve, some object and the others prefer to be neutral. Such a situation cannot be dealt with existing games.

In this study, we define a bicooperative fuzzy game to deal with such a situation. In a bicooperative fuzzy game, we define solution concepts based on the Weber set[2] and the path solution cover[1], and define the catcher. Furthermore, we shall show relations among these solutions.

2. Preliminaries

Let n be a positive finite number, and a set of players is denoted by $N = \{1, 2, ..., n\}$. A cooperative crisp game is defined by an ordered pair (N, cr) where $cr : 2^N \to \mathbb{R}$ with $cr(\emptyset) = 0$. Let $3^N = \{(S, T) : S, T \subseteq N, S \cap T = \emptyset\}$. Then an ordered pair (N, b) where $b : 3^N \to \mathbb{R}$ with $b(\emptyset, \emptyset) = 0$ is called a bicooperative crisp game. Here, for $(S, T) \in 3^N$, $b(S, T)$ represents the proceed when the members of S approve a coalition, the members of T object and the others are neutral.

A fuzzy coalition can be characterized by a vector $s \in [0, 1]^N$. Then the i-th coordinate s_i of s represents the participation level of player i in the cooperative fuzzy coalition s. The set of all fuzzy coalitions are denoted by $\mathcal{F}^N$. For $S \subseteq N$, the cooperative fuzzy coalition $e^S \in \mathcal{F}^N$ is defined by $e^S_i = 1$ if $i \in S$, and $e^S_i = 0$ otherwise. A cooperative fuzzy game is defined by (N, v) where $v : \mathcal{F}^N \to \mathbb{R}$ with $v(e^\emptyset) = 0$.

3. Bicooperative fuzzy games and solution concepts

To introduce a bicooperative fuzzy game, let us define a bicooperative fuzzy coalition by $((s_i, t_i))_{i\in N}$ such that $s_i, t_i \in [0, 1]$ and $s_i + t_i \leq 1$ for any $i \in N$. In stead of $((s_i, t_i))_{i\in N}$, $(s_i, t_i)_{i\in N}$ is also written. s_i and t_i represent the approval level and the objection level of the player i in $(s_i, t_i)_{i\in N}$, respectively. We denote the set of all bicooperative fuzzy coalitions by $\mathcal{BF}^N$. We define a bicooperative fuzzy game by (N, bv) such that $bv : \mathcal{BF}^N \to \mathbb{R}$ with $bv((e^\emptyset_i, e^\emptyset_i)_{i\in N}) = 0$. Let us denote the set of all bicooperative fuzzy games with player set N by $BFG(N)$. A set-valued solution on $BFG(N)$ can be defined by $FBV : BFG(N) \to 2^{\mathbb{R}^N}$.

In this paper, we shall define two types each of the Weber set and of the path solution cover. To introduce two types of the Weber set, let us define a W-path by a sequence $\delta = \langle(s^{\delta,0}_i, t^{\delta,0}_i)_{i\in N}, \ldots, (s^{\delta,m}_i, t^{\delta,m}_i)_{i\in N}\rangle$ of $m+1$ different points in $\mathcal{BF}^N$ satisfying the following;

A: $(s^{\delta,0}_i, t^{\delta,0}_i)_{i\in N} = (e^\emptyset_i, e^N_i)_{i\in N}$, and $(s^{\delta,m}_i, t^{\delta,m}_i)_{i\in N} = (e^N_i, e^\emptyset_i)_{i\in N}$;

B: $s^{\delta,k}_i \leq s^{\delta,k+1}_i$ and $t^{\delta,k}_i \geq t^{\delta,k+1}_i$ for any $i \in N$ and $k = 0, \ldots, m-1$;

C: $\{(s^{\delta,k}_i, t^{\delta,k}_i)_{i\in N} \mid k = 0, \ldots, m\} \subseteq \{(e^S_i, e^T_i)_{i\in N} \mid (S, T) \in 3^N\}$.

Then, $\Delta(N)$ is defined by the set of all W-paths δ in $\mathcal{BF}^N$. We shall define two types of a W-path, which are denoted by W_1-path and W_2-path. Both of these paths represent the process that each player changes his choice from objection to approval. A W_1-path is based on the idea that all players cannot change their actions without passing neutral, while a W_2-path is based on the idea that all players can change directly from objection to approval. A W_1-path is defined by a W-path satisfying

D-1: For each $k \in \{0, \ldots, m-1\}$, there is one player $i \in N$ such that $s^{\delta,k}_i < s^{\delta,k+1}_i$, $t^{\delta,k}_i = t^{\delta,k+1}_i$ or $s^{\delta,k}_i = s^{\delta,k+1}_i$, $t^{\delta,k}_i > t^{\delta,k+1}_i$ while $s^{\delta,k}_j = s^{\delta,k+1}_j$, $t^{\delta,k}_j = t^{\delta,k+1}_j$ for $j \in N\backslash\{i\}$.

Then, $\Delta^1(N)$ is defined by the set of all W_1-paths δ in $\mathcal{BF}^N$.

A W_2-path is defined by a W-path satisfying

D-2: For each $k \in \{0, \ldots, m-1\}$, there is one player $i \in N$ such that $s^{\delta,k}_i \leq s^{\delta,k+1}_i$, $t^{\delta,k}_i \geq t^{\delta,k+1}_i$ and either one of the two inequality holds in the strict sense, while $s^{\delta,k}_j = s^{\delta,k+1}_j$, $t^{\delta,k}_j = t^{\delta,k+1}_j$ for $j \in N\backslash\{i\}$.

Then, $\Delta^2(N)$ is defined by the set of all W_2-paths δ in $\mathcal{BF}^N$.

Let $\Delta_i(\delta) = \{(s^{\delta,k}_i, t^{\delta,k}_i) \mid (s^{\delta,k}_i, t^{\delta,k}_i) \neq (s^{\delta,k+1}_i, t^{\delta,k+1}_i)\}$ for δ. We can define the player i's marginal contribution $x^\delta_i(bv)$ as $x^\delta_i(bv) = \sum_{k:(s^{\delta,k}_i, t^{\delta,k}_i)\in\Delta_i(\delta)}(bv((s^{\delta,k+1}_i, t^{\delta,k+1}_i)_{i\in N}) - bv((s^{\delta,k}_i, t^{\delta,k}_i)_{i\in N}))$ for the W-path δ. For $\Delta'(N) \subseteq \Delta(N)$, we define the Weber set on $\Delta'(N)$ by a function $W : BFG(N) \to 2^{\mathbb{R}^N}$ as the following.

$$W_{\Delta'}(bv) = \mathrm{co}\{x^\delta(bv) \in \mathbb{R}^n \mid \delta \in \Delta'(N)\}.$$

To denote two types of the path solution cover, let us define a Q-path by a sequence δ satisfying A, B and the following;

C': $\{(s^{\delta,k}_i, t^{\delta,k}_i)_{i\in N} \mid k = 0, \ldots, m\} \subseteq \mathcal{BF}^N$.

Then, $\Psi(N)$ is defined by the set of all Q-paths δ in $\mathcal{BF}^N$. We shall define two types of a Q-path, denoted by Q_1-path and Q_2-path. A Q_1-path is based on the idea that all players are not allowed to change their two action levels at the same time, while a Q_2-path is based on the idea that all players can change their two action levels at one time. A Q_1-path is defined by a Q-path satisfying D-1, and $\Psi^1(N)$ is defined by the set of all Q_1-paths δ in $\mathcal{BF}^N$. A Q_2-path is defined by a Q-path satisfying D-2, and $\Psi^2(N)$ is defined by the set of all Q_2-paths δ in $\mathcal{BF}^N$.

Let $\Psi_i(\delta) = \{(s^{\delta,k}_i, t^{\delta,k}_i) \mid (s^{\delta,k}_i, t^{\delta,k}_i) \neq (s^{\delta,k+1}_i, t^{\delta,k+1}_i)\}$ for δ. Then we can define the player i's path solution $x^\delta_i(bv)$ as $x^\delta_i(bv) = \sum_{k:(s^{\delta,k}_i, t^{\delta,k}_i)\in\Psi_i(\delta)}(bv((s^{\delta,k+1}_i, t^{\delta,k+1}_i)_{i\in N}) - bv((s^{\delta,k}_i, t^{\delta,k}_i)_{i\in N}))$ for the Q-path δ. For $\Psi'(N) \subseteq \Psi(N)$, we define the path solution cover on $\Psi'(N)$ by a function $Q : BFG(N) \to 2^{\mathbb{R}^N}$ as the following.

$$Q_{\Psi'}(bv) = \mathrm{co}\{x^\delta(bv) \in \mathbb{R} \mid \delta \in \Psi'(N)\}.$$

Proposition 1: *Let* $bv \in BFG(N)$. $W_{\Delta^1}(bv) \subseteq Q_{\Psi^1}(bv)$ *and* $W_{\Delta^2}(bv) \subseteq Q_{\Psi^2}(bv)$.

4. Conclusion

In this study, we have defined a bicooperative fuzzy game as a new game and solution concepts, the Weber set, the path solution cover and the catcher. Then, We have shown relations among them.

References

[1] Branzei, R., D. Dimitrov, and S. Tijs (2004) : Hypercubes and compromise values for cooperative fuzzy games, *European Journal of Operational Research 155*, 733-740.

[2] Weber, R. (1988) : Probabilistic values for games, in : Roth, A. E. (Ed.), The Shapley Value, *Cambridge University Press*, 101-109.